

# Possible evidence for a change in cosmic equation of state at decoupling

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(19-1-00)

Indirect but significative evidence (from fusion research data, and from the known  ${}^4\text{He}$  abundance) relevant to primordial nucleosynthesis is examined. It is shown that this evidence supports a change in cosmic equation of state at decoupling, compatible with Einstein's cosmological equations, from  $\rho_m T^{-4} \approx \text{const.} (\rho_m = \rho_r)$  prior to decoupling ( $t < t_{af}$ ), to  $\rho_m T^{-3} \approx \text{const.}$  after ( $t > t_{af}$ ), being  $t_{af}$  the atom formation time. Implications at the baryon threshold time (temperature) and beyond are discussed.

PACS numbers: 05.70.Ce, 04.20.Me, 98.80.-k

## I. Introduction

Recent studies of distant Type 1a supernovae indicate that the universe will expand forever [1]. According to them the universe does not have enough matter (visible or dark) to halt the current expansion. The new findings are consistent with an estimate of the time elapsed since the big-bang of the order of  $15 \times 10^9$  years.

The usual (big-bang) approach to cosmic evolution has taken for granted during decades that a single equation of state,  $\rho_m T^{-3} = \text{const.}$ , is capable of describing well cosmic dynamics at all times. Since at the time of atom formation (last scattering) the physical contents of the universe underwent a drastic change from a plasma to a gas of neutral atoms, the validity of the same equation of state prior to and after that event might be questionable.

In this work we examine relevant experimental evidence [2,3] from fusion research data to check whether  $\rho_m T^{-3} = \text{const.}$  ( $\rho_m > \rho_r$ ), where  $\rho_m$  is the matter mass density and  $\rho_r$  the radiation mass density, describes well cosmic evolution between two specific times, the time at nucleosynthesis, happened much earlier than atom formation, and the time at atoms formation. We will show that this experimental evidence supports  $\rho_m T^{-4} = \text{const.}$  ( $\rho_m = \rho_r$ ) from early times to the atoms formation time, followed by  $\rho_m T^{-3} = \text{const.}$  thereafter.

## II. Cosmological equations

We summarize the parametric Friedmann-Lemaitre solutions ( $\Lambda = 0$ ) of Einstein's equations and we put them in a proper form [4] for later use.

For  $k \leq 0$  (open solution):

$$R = R_+ \sinh^2 y, \quad t = (R_+ / |k|^{1/2} c) \{ \sinh y \cosh y - y \} \quad (1)$$

where  $R_+$  is a constant, defined at  $(8\pi G/3)\rho_+ R_+^3 = c^2 |k| R_+$ . From Eq(1),

$$tH \equiv t(\dot{R}/R) = \{ \sinh y \cosh y - y \} \cosh y / \sinh^3 y \geq \frac{2}{3}, \quad (2)$$

$$\Omega \equiv \rho / (3H^2 / 8\pi G) = 1 / \cosh^2 y \quad (3)$$

For  $k \geq 0$  (closed solution):

$$R = R_+ \sin^2 y, \quad t = (R_+ / k^{1/2} c) \{ y - \sin y \cos y \}, \quad (4)$$

and

$$tH = \{ y - \sin y \cos y \} \cos y / \sin^3 y \leq \frac{2}{3} \quad (5)$$

$$\Omega = 1 / \cos^2 y \quad (6)$$

For  $k = 0$  (flat solution),  $tH = 2/3$ ,  $\Omega = 1$ . As it is well known, proponents of inflationary models prefer  $k = 0$ , most observational [5] cosmologists agree that  $0.1 \leq \Omega \leq 0.3$ , and  $\Lambda \neq 0$  is not ruled out.

## III. Equation of state for the transparent universe

There is a growing consensus [6,7] that the time elapsed from the big-bang is  $t_o \simeq 13.7 \times 10^9$  years, and the present value of Hubble's constant is  $H_o \simeq 65 \text{ Km/sMpc}$ , resulting in

$$t_o H_o \simeq 0.910 \pm 0.09 > \frac{2}{3} \quad (7)$$

(in agreement with recent studies of Type 1a supernovae).

This implies an open universe. Through Eq. (2),  $y_o \simeq 1.92$  and

$$\Omega_o \simeq 0.0824 \pm 0.0082 < 1. \quad (8)$$

Due to the fact that at present  $\rho_{mo}$  (matter mass density)  $\gg \rho_{ro}$  (radiation mass density), we get

$$\rho_{mo} \simeq \Omega_o \rho_{co} \simeq \Omega_o (3H_o^2 / 8\pi G) \simeq 6.54 \times 10^{-31} \text{ g/cm}^3. \quad (9)$$

This allows for a relatively moderate amount of dark matter, and even for a certain amount of matter associated to a non-zero cosmological constant, not enough

(by an order of magnitude) to render the universe flat ( $k = 0$ ).

Due to COBE data [8] the present radiation mass density is known with exceptional precision ( $T_o = 2.726 \pm 0.04K$ ). Then

$$\rho_{ro} = \frac{\sigma T_o^4}{c^2} = \frac{(7.63 \times 10^{-15})(2.726)^4}{(3 \times 10^{10})^2} = 4.68 \times 10^{-34} g/cm^3. \quad (10)$$

In the present (transparent) universe the baryon to photon ratio,  $n_b/n_r$ , remains constant in the absence of any substantial interaction between matter and radiation. Therefore the equation of state is determined by

$$\frac{n_b}{n_r} = \frac{\rho_m/m_b}{(\sigma T^4)/(2.8k_B T)} = const.,$$

resulting in

$$\rho_m T^{-3} = \rho_{mo} T_o^{-3} = 3.23 \times 10^{-32} g/cm^3 K^3 \quad (11)$$

This equation of state should be valid at least since the time of atom formation to present. Because  $\rho_m = \rho_{mo}(R_o/R)^3$ , Eq. (11) implies  $RT \approx const.$  since the time of atom formation, and, for  $t < t_o$ , the ratio ( $\rho_r/\rho_m$ ) was about unity at a certain time,  $t_{eq}$ , at which  $\rho_r = \rho_m$  (equality), in the relatively distant past.

#### IV. Coincidence between equality and atom formation times

At  $t = t_{eq}$  (equality), according to Eq.(11), we have

$$\rho_{meq} T_{eq}^{-3} = \left( \frac{\sigma T_{eq}^4}{c^2} \right) T_{eq}^{-3} = \left( \frac{\sigma}{c^2} \right) T_{eq}, \text{ i.e. } T_{eq} = 3808K, \quad (12)$$

which implies also [4] that  $\rho_{meq} = 1.78 \times 10^{-21} g/cm^3$  at

$$t_{eq} = 1.47 \times 10^{13} \text{ sec} = 4.66 \times 10^5 \text{ yrs.} \quad (13)$$

On the other hand at  $t = t_{af}$  (atom formation), Saha's equation for  $x = 1/2$  (i.e. for 50% ionized atoms), right at  $T = T_{af}$ , leads to [4]

$$3.716 \times 10^{-15} = \left( \frac{B_{eff}}{k_B T_{af}} \right)^{5/2} \exp \left[ - \left( \frac{B_{eff}}{k_B T_{af}} \right) \right], \quad (14)$$

where  $B_{eff} = 14.27eV$  and  $\Omega_{po}/\Omega_o = 0.0355/0.0824$  may be used, resulting in an atom formation temperature

$$T_{af} = 3880K, \quad (15)$$

in almost perfect coincidence with  $T_{eq} = 3808K$ , given above.

This coincidence is remarkable and might suggest that  $(\rho_r/\rho_m) = 1$  (equality) was constant all the way from

well prior to atom formation and began to decrease towards its present value  $(\rho_{ro}/\rho_{mo}) \approx 7.15 \times 10^{-4}$  just when the universe became transparent (we have used the values for  $\rho_{mo}$  and  $\rho_{ro}$  given by Eqs. (9) and (10) respectively). This could mean that, preceding the present matter dominated epoch, the universe might have gone through an epoch of matter/radiation **equality**, rather than radiation **domination**, as currently assumed.

We will come back to this point after considering the evidence provided by fusion research data and  $^4He$  abundance.

#### V. Fusion research data

Fusion at the sun's center (complete) and primordial cosmic fusion (incomplete) admittedly take place under different physical conditions, in particular density. However the threshold fusion temperature is probably well represented by current fusion research data. In what follows we will make first a rough estimate of  $\rho_{mns} T_{ns}^{-3}$  at cosmic nucleosynthesis to be compared with  $\rho_{af} T_{af}^{-3}$  at atom formation, and we will see that the data used suggest an equation  $\rho T^{-n} \sim constant$  with  $n$  different from  $n = 3$ . Later we will make a more refined argument using the well known  $^4He$  abundance and using an equivalent alternative equation of state involving  $t$  (time) and  $T$  (temperature).

The temperature and plasma density at the center of the sun [9], where  $^4He$  nucleosynthesis is taking place continuously, may be expected to be within a factor of the order of a few orders of magnitude of the cosmic nucleosynthesis temperature and density. They are given by

$$T_{\odot}(ctr) \simeq 1.6 \times 10^7 K, \quad \rho_{\odot}(ctr) \simeq 150 g/cm^3. \quad (16)$$

But these data are probably not directly applicable to our problem, because in the sun complete nucleosynthesis is produced.

Fusion research data [2,3], on the other hand, indicate that  $^4He$  synthesis is ignited (or stops) at an ion temperature given by

$$T_{io} = 30keV = 3.47 \times 10^8 K. \quad (17)$$

What are the implications of these data for primordial nucleosynthesis?

Taking  $\rho_{mns} \simeq 0.5 g/cm^3$ ,  $T_{ns} \simeq T_{io}$  and introducing them in Eq. (11) one gets

$$\rho_{mns} T_{ns}^{-3} = 1.04 \times 10^{-27} g/cm^3 K^3, \quad (18)$$

much larger than

$$\rho_{mo} T_o^{-3} = \rho_{maf} T_{af}^{-3} = 3.23 \times 10^{-32} g/cm^3 K^3, \quad (19)$$

which should be expected in the absence of change in the equation of state.

On the other hand if one lets  $\rho_m T^{-n} = \text{const.}$ , with  $n$  free to match  $\rho_{mns} T_{ns}^{-n}$  and  $\rho_{af} T_{af}^{-n}$ , one gets

$$n = \frac{\ln(\rho_{mns}/\rho_{af})}{\ln(T_{ns}/T_{af})} = 4.12 \quad (20)$$

which is compatible with  $n=4$  and therefore, with  $\rho_m = \rho_r = \sigma T^4/c^2$  prior to decoupling (atom formation). This implies

$$\begin{aligned} \rho_m T^{-4} &= \sigma/c^2 = 8.47 \times 10^{-36} \text{ g/cm}^3 \text{ K}^4 \sim \\ \rho_{mns} T_{ns}^{-4} &= 3.44 \times 10^{-35} \text{ g/cm}^3 \text{ K}^4 \end{aligned} \quad (21)$$

The agreement now is not perfect, because there is an indeterminacy in  $\rho_{mns}$ , but it is certainly much better than that between Eqs. (18) and (19).

As shown below, However, we can make use of the  ${}^4\text{He}$  cosmic abundance to refine the experimental estimate of  $n$  in the equation of state prior to decoupling.

## VI. ${}^4\text{He}$ cosmic abundance and the time at nucleosynthesis

The neutron to proton ratio ( $n/p$ ), which is determined by the  ${}^4\text{He}$  abundance at nucleosynthesis and became frozen thereafter [10] is given by

$$(n/p) \simeq \frac{1}{2} Y_P / (1 - \frac{1}{2} Y_P) \simeq 0.131, \quad (22)$$

( $Y_P = 0.232 \pm 0.008$ ), combined with the neutron lifetime

$$\tau_n = 14.78 \text{ min} = 887 \text{ sec.}, \quad (23)$$

leads to

$$\begin{aligned} (n/p) &\simeq e^{-t_{ns}/\tau_n} \text{ i.e.} \\ t_{ns} &= t(T_{ns}) = 1803 \text{ sec.} \end{aligned} \quad (24)$$

For  $y \ll 1$  the cosmological equations reduce to

$$t = (R_+ / |k|^{1/2} c) \frac{2}{3} y^3, \quad y = (\rho/\rho_+)^{-1/6} \quad (25)$$

This, with  $\rho T^n \sim \text{const.}$ , entails  $t T^{n/2} = \text{const.}$

At  $t = t_{af} = 1.47 \times 10^{13} \text{ sec}$  and  $T = T_{af} = 3808$ , from which, with  $n = 3$ , we get

$$t_{af} T_{af}^{3/2} = 3.45 \times 10^{18} \text{ sec K}^{3/2}, \quad (26)$$

which is way off

$$t_{ns} T_{ns}^{3/2} = 1.16 \times 10^{16} \text{ sec K}^{3/2}, \quad (27)$$

Letting  $n$  free in  $t T^{n/2} \simeq \text{const}$  (equivalent to  $\rho T^{-n} \simeq \text{const.}$ ), on the other hand, we get

$$n = 2 \frac{\ln(t_{af}/t_{ns})}{\ln(T_{ns}/T_{af})} = 3.99 \quad (28)$$

which gives a perfect agreement with  $n = 4$ . Now we get

$$t_{af} T_{af}^2 = 2.1 \times 10^{20} \simeq t_{ns} T_{ns}^2 = 2.1 \times 10^{20} \text{ sec K}^2 \quad (29)$$

This is consistent with  $\rho_{ns} = \rho_{af} (t_{af}/t_{ns})^2 \simeq 0.125 \text{ g/cm}^3$ , about one fourth of the value used above.

Thus fusion data and  ${}^4\text{He}$  cosmic abundance data appear to reinforce each other in support of an equation of state prior to decoupling

$$\rho_m T^{-4} \simeq \rho_{af} T_{af}^{-4} \simeq 8.47 \times 10^{-36} \text{ g/cm}^3 \text{ K}^4 \quad (t < t_{af}) \quad (30)$$

We may note that other light element abundances [11], ( $D$ ,  $B$ ,  $Be$ ,  ${}^7\text{Li}$ ), very minute in comparison with the  ${}^4\text{He}$  abundance, and always open to interpretation [12], are not considered here.

Let us go one step further and investigate physical conditions at the cosmic threshold for baryons and beyond.

## VII. Baryon threshold and beyond

Primordial elementary particles cannot be said to have individual existence before the cosmic density  $\rho_m$  was equal to the particle density  $\rho_z \simeq m_z / \frac{4\pi}{3} r_z^3$ , where  $m_z$  is the mass of the particle and  $r_z$  its radius. The threshold temperature for  $\rho_m \simeq \rho_z$  is given by Stephan-Boltzmann's law as  $T_z = m_z c^2 / 2.8 k_B$ .

For baryons (protons, neutrons)  $m_b = 1.67 \times 10^{-24} \text{ g}$  and  $r_b \simeq 1.2 \times 10^{-13} \text{ cm}$  (from scattering experiments). With this data we get  $\rho_b = 2.0 \times 10^{14} \text{ g/cm}^3$  and  $T_b = 3.88 \times 10^{12} \text{ K}$  which gives

$$\begin{aligned} \rho_b T_b^{-3} &= 3.42 \times 10^{-24} \text{ g/cm}^3 \text{ K}^3 >> \\ \rho_{maf} T_{af}^{-3} &= 3.23 \times 10^{-32} \text{ g/cm}^3 \text{ K}^3 \end{aligned} \quad (31)$$

On the other hand

$$\begin{aligned} \rho_b T_b^{-4} &= 8.82 \times 10^{-37} \text{ g/cm}^3 \text{ K}^4 \sim \\ \rho_{maf} T_{af}^{-4} &= 8.82 \times 10^{-36} \text{ g/cm}^3 \text{ K}^4 \end{aligned} \quad (32)$$

which is in relatively good order of magnitude agreement with an equation of state  $\rho_m T^{-4} \sim \text{const.}$  ( $\rho_m = \rho_r$ ) prior to decoupling (opaque universe). With this equation the baryon to photon ratio at  $T = T_b$  is

$$\frac{n_b(T_b)}{n_r(T_b)} = \frac{\rho_b/m_b}{(\rho_r c^2)/2.8 k_B T_b} = \frac{T_b}{m_b c^2 / 2.8 k_B} = 1 \quad (33)$$

being, consequently,  $(n_b/n_r)_{T < T_b}$  less than one, and  $(n_b/n_r)_{T > T_b}$  more than one. At  $T_{eq} = 3808 \text{ K}$  (atom formation/decoupling), using Eq. (12) and (13), the baryon/photon ratio becomes

$$\frac{n_b(T_{af})}{n_r(T_{af})} = \frac{\rho_{meq}/m_b}{(\sigma T_{eq}^4)/2.8 k_B T_{eq}} = 9.77 \times 10^{-10} \quad (34)$$

and, as noted in section III, it remains constant thereafter up to the present epoch and beyond, as long as the universe remains transparent. In other words, prior to decoupling, with the new equation of state (applicable to an opaque universe), the baryon to photon ratio decreases as times goes on, and the background temperature decreases accordingly.

In the meantime radiation pressure is continuously pushing matter radially, until the time of last scattering (atom formation). At this time the universe becomes transparent, radiation pressure stops, protogalaxies begin to form, and their constituent atoms, having already gained a tremendous momentum from the background radiation in the plasma (opaque) phase of the expansion, departed from each other at enormous speed. After decoupling (atom formation) the universe has become transparent, the baryon to photon ratio has become frozen, and consequently  $(n_b/n_r)_{T_o} = (n_b/n_r)_{T_{af}}$ .

For Planck's monopoles  $m_p = (ch/G)^{1/2} = 5.4 \times 10^{-5}g$  and  $r_p = (Gh/c^3)^{1/2} = 4.0 \times 10^{-33}cm$ . With these numbers we obtain  $\rho_p = 2.0 \times 10^{92}g/cm^3$  and  $T_p = 3.5 \times 10^{32}K$ , which result in

$$\rho_p T_p^{-4} = 1.33 \times 10^{-38} \sim \rho_{maf} T_{af}^{-4} = 8.82 \times 10^{-36}g/cm^3, \quad (35)$$

again in relatively good order of magnitude agreement with an equation of state  $\rho_m T^{-4} \sim const.$  ( $\rho_m = \rho_r$ ). Cosmic time at this temperature, by means of Eq. (29) is given by

$$t_p = t_{af} (T_{af}/T_p)^2 = 1.74 \times 10^{-45} \sim (Gh/c^5)^{1/2} = 1.33 \times 10^{-43} \text{ sec}. \quad (36)$$

We can go a final step back in time. The earliest conceivable time determined by Heisenberg's uncertainty principle for a universe of mass  $M \sim \rho_+ \frac{4\pi}{3} R_+^3 \sim 5 \times 10^{55}g$  is

$$t = t_L = h/Mc^2 \simeq 1.46 \times 10^{-103} \text{ sec}. \quad (37)$$

This time can be properly called Lemaitre's time ( $t_L$ ), and a single particle of this mass, a Lemaitre monopole (the "primordial atom" or "primitive egg" of George Lemaitre). The Compton radius of this monopole would be  $r_L = h/Mc = 4.4 \times 10^{-93}cm$ . With these numbers we get  $\rho_L = 4.3 \times 10^{333}g/cm^3$  and  $T_L = 1.1 \times 10^{92}K$ , resulting in

$$\rho_L T_L^{-4} = 2.9 \times 10^{-35} \sim \rho_{maf} T_{af}^{-4} = 8.82 \times 10^{-36}g/cm^3, \quad (38)$$

which is not in bad order of magnitude agreement with our equation of state for the opaque universe prior to decoupling.

## VIII. Concluding remarks

As we have seen, the gross features of cosmic dynamics seem to be reasonably well described with a twofold equation of state

$$\rho_m T^{-4} = \sigma/c^2 = 8.47 \times 10^{-36}g/cm^3 K^4, \text{ at } t < t_{af} \quad (39)$$

and

$$\rho_m T^{-3} = (\sigma/c^2) T_{af} = 3.23 \times 10^{-32}g/cm^3 K^3 \text{ at } t > t_{af} \quad (40)$$

being  $t_{af} = 1.47 \times 10^{13} \text{ sec} = 0.466 \times 10^6 \text{ yrs}$  and  $T_{af} = 3808K$ , respectively, the time and temperature for atom formation (decoupling). After decoupling,  $\rho_m T^{-3} = const.$  is the standard, currently accepted equation of state. Prior to decoupling  $\rho T^{-4} \simeq const.$  implies a drastic departure [13] from standard usage.

Eqs. (39) and (40) contain only universal constants and  $T_{af}$ , directly related to  $\hbar\omega_{af} = 2.8k_B T_{af}$ , the energy at which  $H$  atoms (the main constituents of the universe) begin to form, as given by Saha's law, Eq. (14).

The proposed change may be unconvincing or even wrong. But one think it is: simple.

One of us (J.A.G.) would like to thank S.L.Jaki for historical perspective on the subject, and J.C.Mather and A.Hewish for correspondence and friendly criticism of previous versions of this work. We thank H.Schulz for pointing out to us more accurate physical data at the sun's center.

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